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ROBUST STABILIZATION OF DYNAMICAL SYSTEMS BY MEANS OF BOUNDED CONTROLS†

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The problem of stabilizing linear systems whose parameters which are known with a finite accuracy (robust stabilization) is considered. Optimal control methods, which enable one to obtain positional solutions of special auxiliary optimal control problems, are used to construct bounded stabilizing feedbacks. The implementation of the proposed stabilization methods depends very much on the capabilities of modern computational technology. The results are illustrated by taking the robust stabilization of third- and fourth-order dynamical systems as examples.

In the majority of papers which have been published on robust stabilization, either the structure of the stabilizing feedback is specified, constraints on the stabilizing actions are ignored or both of these apply at the same time. It is clear that artificial constraints on the structure of the feedbacks and the neglect of geometric constraints on the control are hardly in accord with present-day requirements for control systems. A method for stabilizing deterministic systems using optimal control theory was therefore proposed in [1, 2] in which the above-mentioned requirements are fully taken into account. The aim of this paper is to extend this method to the problem of robust stabilization.

1. FORMULATION OF THE PROBLEM

Consider a dynamical system, the behaviour of which, when $t \geqslant 0$, is described by the equation

$$\dot{x} = Ax + bu \tag{1.1}$$

$$(x, b \in R^n, u \in R, A \in R^{n\times n}, \operatorname{rank}(b, Ab, \ldots, A^{n-1}b) = n)$$

where $x = x(t)$ is the n-vector of the state of the system at the instant of time t and $u = u(t)$ is the value of the scalar control.

We assume that the available information on the system parameters A and b is inexact: the $n \times n$ matrix A and the n-vector b are such that

$$A = A_0 + \Delta A, \quad b = b_0 + \Delta b$$

where A_0, b_0 are the known $n \times n$ matrix and the n-vector, respectively, and ΔA, Δb are the unknown $n \times n$ matrix and the n-vector which satisfy the equations

$$\|\Delta A\| \leqslant \alpha, \quad \|\Delta b\| \leqslant \beta \quad (\alpha, \beta > 0)$$

Suppose G is a bounded neighbourhood of the equilibrium state $x = 0$ of system (1.1) and $u = 0$. For a fixed $\varepsilon > 0$ and fixed numbers $\nu > 0$, $L > 0$, we call the function

$$u(t, x), \; x \in G, \; t \in [0, \nu[\tag{1.2}$$

the bounded robust stabilizing preset-position control (PPC) of system (1.1) in the domain G if

1. $u(t, 0) = 0, \; t \in [0, \nu[$;
2. $|u(t, x)| \leqslant L, \; x \in G, \; t \in [0, \nu[$;

3. the trajectory of the closed system

$$\dot{x} = Ax + bu(t, x), \quad x(0) = x_0, \quad x_0 \in G \tag{1.3}$$

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is a continuous solution of the equation

$$\dot{x} = Ax + bu(t), \quad x(0) = x_0$$

when $u(t) = u(t - k\nu, x(k\nu)), t \in [k\nu, (k+1)\nu[, k = 0, 1, \ldots$;

4. system (1.3) is asymptotically stable in G when $A \equiv A_0, b \equiv b_0$;

5. a number $t(\varepsilon) > 0$ exists such that each solution $x(t), t \geqslant 0$ of system (1.3) satisfies the conditions $\| x(t) \| \leqslant \varepsilon, \geqslant t(\varepsilon)$.

The explicit (formula) construction of the stabilizing function (1.2), which satisfies the above-mentioned requirements in the case of a sufficiently large domain G, is outside limits of the possibilities of the mathematical methods used in modern control theory. The aim of the following exposition is to show that a roundabout method of individual robust stabilization exists, which is based on computational techniques. The basic idea behind the proposed method consists of the introduction of an auxiliary (associated) optimal control problem, the construction of a preset-positional solution for this problem, proof of the stabilizing properties of this solution and the realization of the preset-positional solution of the associated problem using an optimal regulator. It will be shown that an algorithm for the operation of the optimal regulator, which is executed using modern microprocessors, can be obtained for quite complex systems and that the signals which are generated by the regulator achieve robust stabiliztion. In this paper, this regulator is therefore called the stabilizer.

2. THE ASSOCIATED OPTIMAL CONTROL PROBLEM. THE STABILIZING PROPERTY OF OPTIMAL FEEDBACK

We choose the natural numbers N, m $(N > m > n)$ and the real number $h > 0$ and suppose that $\nu = mh, \Theta = Nh$.

We shall call a piecewise-constant function $u(t), t \in T = [0, \Theta]$: $u(t) = u_j, t \in [(j-1)h, jh[, j = 1, 2, \ldots, N$ which satisfies the constraint $| u(t) | \leqslant L, t \in T$, an accessible control.

We shall say that an accessible control $u(t), t \in T$ is permissible for a state $z \in R^n$ if the trajectory $x(t), t \in T$ of the system

$$\dot{x} = A_0 x + b_0 u, \quad x(0) = z$$

satisfies the condition $x(\Theta) = 0$.

We shall evaluate the quality of a permissible control using the value of the functional

$$\rho(u) = \max_{t \in [0, \Theta]} | u(t) |$$

A permissible control $u^0(t) = u^0(t | z), t \in T$ is referred to as the optimal program control for a state $z \in R^n$ if the quality criterion attains the minimum value $\rho(u^0) = \min \rho(u)$ in it, where the minimum is taken over all permissible controls.

The optimal program is therefore the solution of the problem

$$\begin{gathered} \rho(z) = \min \rho \\ \dot{x} = A_0 x + b_0 u, \quad x(0) = z \\ x(\Theta) = 0, \quad |u(t)| \leqslant \rho, \quad t \in T \end{gathered} \tag{2.1}$$

We shall assume that the condition

$$\operatorname{rank}(b_{0h}, A_{0h} b_{0h}, \ldots, A_{0h}^{n-1} b_{0h}) = n \tag{2.2}$$

$$\left(A_{0h} = \exp A_0 h, \quad b_{0h} = b_0 \int_0^h \exp s A_0 ds \right)$$

is satisfied in the case of problem (2.1).

The optimal starting preset-position control (PPC) is defined by the equality

$$u^0(t, z) = u^0(t | z), \quad t \in [0, \nu[, \quad z \in R^n$$

We now introduce the set

$$G_\Theta = \{z \in R^n : |u^0(t,z)| \leqslant L, t \in [0, \nu]\}$$

It possesses the property: a $\Theta > 0$ exists for any $\varepsilon > 0$ such that all states of system (1.1) which can be transferred by permissible controls to the origin of coordinates after a finite time are contained in an ε-neighbourhood of the set G_Θ.

We shall show that, for specified ε, G, α, β and an appropriate choice of the parameters $\Theta > 0$, $\nu > 0$, $h > 0$ of problem (2.1), the feedback

$$u(t,x) = u^0(t,x), \quad x \in G_\Theta, \quad t \in [0, \nu[$$

will satisfy all the requirements for determining a bounded robust stabilizing PPC with $G = G_\Theta$.

The properties listed below follow from the definition of the starting PPC:

1. $u(t, 0) \equiv 0, t \in [0, \nu[$;
2. $|u(t,x)| \leqslant L, x \in G, t \in [0, \nu[$;
3. the trajectory of the closed system (1.3) when $u(t,x) = u^0(t,z)$ is the continuous solution of the system of equations

$$\dot{x} = A_0 x + b_0 u^0(t), \quad x(0) = x_0$$
$$u^0(t) = u^0(t - k\nu, x(k\nu)), \quad t \in [k\nu, (k+1)\nu[, \; k = 0, 1, \ldots$$

where $u^0(t-k\nu, x(k\nu)) = u^0(t-k\nu\,|x(k\nu))$, $t \in [0, \nu[$; $u^0(t\,|x(k\nu))$, $t \in [0, \Theta]$ is the optimal preset control of problem (2.1) for the state $z = x(k\nu)$.

We shall show, by the method of Lyapunov functions [3, 4], that system (1.1) when $A \equiv A_0$, $b \equiv b_0$, that is, the system

$$\dot{x} = A_0 x + b_0 u^0(t,x) \tag{2.3}$$

is asymptotically stable in G_Θ.

The optimum value of the performance criterion (PC) $\rho(z)$, $z \in G_\Theta$ of problem (2.1) is taken as a Lyapunov function. Suppose that, at an arbitrary actual instant of time $\tau = l\nu$, system (2.3) is in a state $x^*(\tau\,|x_0^*)$ which corresponds to an arbitrary initial state $x(0) = x_0^*, x_0^* \in G_\Theta$. In the actual state $x^*(\tau\,|x_0^*)$, the PC of problem (2.1) takes the value $\rho(x^*(\tau\,|x_0^*))$, which is calculated by solving (2.1) with $z = x^*(\tau\,|x_0^*)$.

Using Cauchy's formula, we eliminate the variable states from problem (2.1) and write it, taking account of the class of permissible controls used, in the equivalent functional form

$$\begin{gathered}\rho \to \min \\ F_0(\Theta)x^*(\tau\,|\,x_0^*) + \sum_{j=1}^{N} u_j \int_{(j-1)h}^{jh} F_0(\Theta - t) b_0 dt = 0 \\ |u_j| \leqslant \rho, \; j = 1, 2, \ldots, N\end{gathered} \tag{2.4}$$

Here, $F_0(t)$, $t \geqslant 0$ is the fundamental matrix of the solutions of the homogeneous system $\dot{x} = A_0 x$ ($\dot{F}_0 = A_0 F_0$, $F_0(0) = E$).

We now reduce problem (2.4) to another equivalent problem of linear programming by introducing the new variables $\xi_0 = 1/\rho$, $\xi_j = u_j/\rho$, $j = 1, 2, \ldots, N$

$$\begin{gathered}\xi_0 \to \max \\ F_0(\Theta)x^*(\tau\,|\,x_0^*)\xi_0 + \sum_{j=1}^{N} \xi_j \int_{(j-1)h}^{jh} F_0(\Theta - t) b_0 dt = 0 \\ \xi_0 \geqslant 0, \; |\xi_j| \leqslant 1, \; j = 1, 2, \ldots, N\end{gathered} \tag{2.5}$$

The following notation is used: $\xi_\tau^0(\cdot) = (\xi_j^0(\tau), j = 0, 1, \ldots, N)$ is the optimal plan and $K^0(\tau)$ is the optimal support [5] of problem (2.5). The optimal control of problem (2.4) will then take the form $u_\tau^0(\cdot) = (u_j^0(\tau) = \xi_j^0/\xi_0^0, j = 1, 2, \ldots, N)$.

At the instant of time $\tau + \nu$, system (2.3) under the action of the control $u_j^0(\tau), j = 1, 2, \dots, m$ is in the state

$$x^*(\tau+\nu \mid x_0^*) = F_0(\nu)x^*(\tau \mid x_0^*) + \sum_{j=1}^{m} u_j^0(\tau) \int_{(j-1)h}^{jh} F_0(\nu - t)b_0 dt$$

In the case of the state $z = x^*(\tau + \nu \mid x_0^*)$, the PC of problem (2.1) takes the value $\rho(x^*(\tau + \nu \mid x_0^*))$. The inequality

$$\rho(x^*(\tau+\nu \mid x_0^*)) < \rho(x^*(\tau \mid x_0^*)) \tag{2.6}$$

holds.

In fact, the control $u_{\tau+\nu}(\cdot) = (u_{j+m}^0(\tau), j = 1, 2, \dots, N - m;\ u_j^0 = 0, j = N - m + 1, N - m + 2, \dots, N)$ is a permissible control for the state $z = x^*(\tau + \nu \mid x_0^*))$ and the PC of problem (2.1) for this control satisfies the inequality

$$\rho(x^*(\tau+\nu \mid x_0^*)) \leqslant \rho(x^*(\tau \mid x_0^*)) \tag{2.7}$$

This means that inequality (2.7) will also be satisfied for the optimal control $u_{\tau+\nu}^0(\cdot)$.

We now consider problem (2.5) in the interval $[\tau, \tau + \nu]$ for the state $x^*(\tau + \nu \mid x_0^*)$ (Problem A).

The set $\xi^- = (\xi_0^- = \xi_0^0(\tau), \xi_j^- = \xi_j^0(\tau), j = 1, 2, \dots, N, \xi_0^-, j = N + 1, N + 2, \dots, N + m)$ is a plan of problem A for which the value of the PC is equal to $\xi_0^- = \xi_0^0(\tau)$.

We now construct a new support $K_s(\tau + \nu)$ of problem A using the optimal support $K^0(\tau)$ of problem (2.5). The optimal plan $(\xi^-, K_s(\tau + \nu)]$ is non-degenerate. It is indispensable to find the non-zero estimates among the estimates $\Delta_j, j = N + 1, N + 2, \dots, N + m$ since the identity $\Delta_j \equiv 0, j = N + 1, N + 2, \dots, N + m$ contradicts condition (2.2). The inequality $\xi_0^0(\tau + \nu) > \xi_0^0(\tau)$ will therefore be satisfied for the optimal plan of problem A. This means that inequality (2.6) holds.

Using inequality (2.6), it can be shown that $\rho(x^*(t \mid x_0^*)) \to 0$, $\| x^*(t \mid x_0^*) \| \to 0$ when $t \to \infty$.

We will now prove property 5 for determining the bounded robust stabilizing PPC.

The arbitrary actual instant of time $\tau = l\nu$ and the state $x^*(\tau) = x^*(\tau | x^*_0)$ of system (1.1), which corresponds to an arbitrary initial state $x(0) = x_0^*, x_0^* \in G_\Theta$ and which has been realized by the values A^*, b^* of the parameters A and b of system (1.1), are now considered.

We introduce the sets

$$X_\nu = \left\{ x \in R^n :\ x(\nu) = F(\nu)x^*(\tau \mid x_0^*) + \int_0^\nu F(\nu - t)bu^0(t \mid x^*(\tau))dt,\ \|\Delta A\| \leqslant \alpha,\ \|\Delta b\| \leqslant \beta \} \right\}$$

$$X_\Theta = \left\{ x \in R^n : x(\Theta) = F_0(\Theta - \nu)x(\nu) + \int_\nu^\Theta F_0(\Theta - t)b_0 u^0(t \mid x^*(\tau))dt \right\}$$

$$X_0 = \left\{ x \in R^n : F_0(\nu)x + \int_\nu^0 F_0(\nu - t)b_0 u(t)dt = 0,\ | u(t) | \leqslant \rho,\ t \in [0, \nu[\right\}$$

where $F(t), t \geqslant 0$ is the fundamental matrix of the solutions of the homogeneous system $\dot{x} = Ax$.

The minimum value of ρ, for which the condition $X_0 \supset X_\Theta$ is satisfied, is denoted by ρ^*.

Suppose that, for a state $x^*(\tau \mid x_0^*)$, the optimal value of the PC of problem (2.1) with $z = x^*(\tau \mid x_0^*)$ satisfies the inequality $\rho(x^*(\tau \mid x_0^*)) > \rho^*$. We denote by $u_\tau^0(\bullet) = (u_j^0(\tau) = u_j(x^*(\tau \mid x_0^*)) = \xi_j^0/\xi_0^0$, $j = 1, 2, \dots, N)$ the optimal plan of problem (2.4) with $z = x^*(\tau \mid x_0^*)$, and by $\xi_\tau^0(\bullet) = (\xi_j^0, j = 0, 1, \dots, N)$ and $K^0(\tau)$ the optimal plan and optimal support of problem (2.5), respectively, corresponding to the initial state $z = x^*(\tau \mid x_0^*)$.

At the instant of time $z = x^*(\tau \mid x_0^*)$, the closed system (1.3) is in the state

$$x^*(\tau+\nu) = x^*(\tau+\nu \mid x_0^*) = F(\nu)x^*(\tau) + \sum_{j=1}^{m} u_j^0(\tau) \int_{(j-1)h}^{jh} F(\nu - t)bdt$$

for which the PC of problem (2.1) takes the value $\rho(x^*(\tau+\nu))$.

The inequality

$$\rho(x^*(\tau+\nu)) < \rho(x^*(\tau)) \tag{2.8}$$

holds for all $x^*(\tau)$ for which

$$\rho(x^*(\tau)) > \rho^* \tag{2.9}$$

Actually, according to the definition of the number ρ^*, a control $v(t)$, $|v(t)| \leqslant \rho^*$, $t \in [\tau + \Theta, \tau + \Theta + \nu[$ exists such that the control $u_{\tau+\nu}(\bullet) = (u^0_{j+m}(\tau), j = 1, 2, \ldots, N-m, u^0_j(\tau) = v_{j-N+m}$, is a permissible control for problem (2.4) with an initial state $z = x^*(\tau + \nu)$ and the inequality

$$\max_{j=1,2,\ldots,N} |u_j(\tau+\nu)| \leqslant M$$

$$M = \max\left(M^0, \max_{j=N-m+1,\ N-m+2,\ldots,N} |v_{j-N+m}(\tau)|\right)$$

$$M^0 = \max_{j=1,2,\ldots,N-m} |u^0_j(\tau)|$$

is satisfied for it.

The inequality

$$\rho(x^*(\tau+\nu)) = \max_{j=1,2,\ldots,N} |u^0_j(\tau+\nu)| \leqslant M^0$$

will be satisfied for the optimal control $u^0\tau + \nu(\bullet)$.

Since $M = M^0 = \rho(x^*(\tau))$, the non-strict inequality (2.8) follows from this.

It follows from the inequality (2.9) that the optimum value of the performance criterion is determined by the minimum intensity of the control $u^0_j(\tau)$, $j = 1, 2, \ldots, N-m$ which is constructed according to the deterministic problem. Inequality (2.8) for the state $x^*(\tau + \nu)$ therefore follows from the proof of Property 4 in the determination of the stabilizing preset-position control.

Using the definition of the set X_Θ, it can be shown that for any bounded numbers α, $\beta > 0$ and any number $\varepsilon_1 > 0$ which may be as small as desired, numbers $\nu > 0$, $\Theta > 0$ can be found such that $\|x(t)\| \leqslant \varepsilon_1$ for $x \in X_\Theta$.

Consequently, $\Theta > 0$, $\nu > 0$, $t_1(\varepsilon)$ can be chosen such that the Lyapunov function $\rho(x)$, $x \in G$ will decrease along the trajectories of system (1.3) for all states which lie outside an ε-neighbourhood, which may be as small as desired, of the equilibrium state $x = 0$ and $\|x(t)\| \leqslant \varepsilon$ when $t \geqslant t_1(\varepsilon)$.

3. AN ALGORITHM FOR THE OPERATION OF THE OPTIMUM STABILIZER

Prior to the functioning of the system at the instant of time $\tau = 0$, the stabilizer constructs the optimal preset control $u^0_0(\bullet)$ of problem (2.1) for the state $z = x^*_0$. This can be done, for example, by the methods of linear programming [5], since all the elements of the problem are known. If the state x^*_0 is previously unknown but the domain in which it can appear is known, then this domain can be covered with a finite net and problem (2.1) can be solved in advance at the mesh points of this net. The solution at the instant of time $\tau = 0$ for the state x^*_0 which has been realized is obtained by correction of one of the solutions obtained using the method described below for arbitrary τ.

We assume that the stabilizer works at the instants of time $0, \nu, \ldots, (l-1)\nu$ and, at the instants of time $\tau = l\nu$, system (1.1) is found to be in the state $x^*(\tau)$. By assumption, the stabilizer already knows the solution of problem (2.1) at the instant of time $\tau - \nu$. This problem is equivalent to a problem in linear programming (Problem S) which differs from problem (2.5) solely in the vector of conditions accompanying the variable ξ_0, and this difference is smaller the smaller the value of ν.

In order to work out the control $u^*(t)$, $t \in [\tau, \tau + \nu[$, it is necessary that the stabilizer knows the solution of problem (2.1) with the initial condition $z = x^*(\tau)$, that is, the solution of problem (2.5).

According to the theory of linear programming [6], the dual method is the most effective for solving Problem S since, in this case, the optimal support $K^0_s(\tau)$ of problem (2.5) is constructed after a small number of iterations and the optimal support $K^0_s(\tau - \nu)$ of problem S is taken as the initial support. If the time required by the specific computer to construct, at the instant of time τ, a new support $K^0_s(\tau)$ using the support $K^0_s(\tau - \nu)$ already constructed at the preceding instant of time $\tau - \nu$, is less than

ν, it can be said that, in the case of the problem considered, the robust stabilizing feedback (1.2) can be realized in real time using the chosen computational device [7, 8]. Since, within the framework of linear programming methods, the dimensions of problem S are not large, while the operating speed of present-day microprocessors is very high, the method of stabilization which has been described can be used in the case of dynamical systems of quite high order.

4. EXAMPLES

As the first example we will consider the problem of the damping, by bounded piecewise-continuous controls, of the oscillations in a two-mass system (Fig. 4).

The mathematical model of such a system has the form

$$\begin{aligned} &\dot{x}_1 = x_3, \ \dot{x}_2 = x_4 \\ &\dot{x}_3 = (-c_1x_1 + c_2x_2 + u)/m, \ \dot{x}_4 = (c_1x_1 - (c_1 + c_2)x_2)/M \end{aligned} \tag{4.1}$$

where m and M are the masses of the objects, c_1 and c_2 are the coefficients of elasticity of the springs and u is the damping action.

Suppose that, at the initial instant of time $t = 0$, the system under consideration is found to be in a state $x_1(0) = 0.5, x_2(0) = 0.4, x_3(0) = 0.2, x_4(0) = -0.1$. It is required that the oscillations of the system should be quenched.

The robust stabilizer was calculated for the following nominal values of the system parameters: $m_0 = 1$, $M = 10, c_{10} = 1, c_{20} = 9.2$. It was assumed in the calculations that $\Theta = 8, \nu = 2, h = 0.2$.

In this case, the associated problem has the form

$$\begin{aligned} &\rho \to \min \\ &\dot{x}_1 = x_3, \ \dot{x}_2 = x_4 \\ &\dot{x}_3 = -x_1 + 9{,}2x_2 + u, \ \dot{x}_4 = 0{,}1x_1 - 1{,}02x_2 \\ &x_1(0) = x_1^*(\tau), \ x_2(0) = x_2^*(\tau), \ x_3(0) = x_3^*(\tau), \ x_4(0) = x_4^*(\tau) \\ &x_1(\Theta) = 0, \ x_2(\Theta) = 0, \ x_3(\Theta) = 0, \ x_4(\Theta) = 0 \\ &|u(t)| \leqslant \rho, \ t \in T = [0, \Theta] \end{aligned}$$

where $x^*(\tau) = (x_1^*(\tau), x_2^*(\tau), x_3^*(\tau), x_4^*(\tau))$ is the state of system (4.1) at the actual instant of time τ.

The following values of the system parameters were realized during the operation of the stabilizer:

1. the coefficient of elasticity, c_1, of the upper spring (Fig. 2) was varied (the dashed curves correspond to the nominal value $c_{10} = 1$ and the solid curves 1 and 2 correspond to the realized values of $c_1^* = 0.9$ and $c_1^* = 1.1$);
2. the coefficient of elasticity, c_2, of the lower spring (Fig. 3) was varied (the dashed curves correspond to the nominal value $c_{20} = 9.2$ and the solid curves 1 and 2 correspond to the realized values $c_2^* = 8.28$ and $c_2^* = 10.12$);
3. the mass m of the upper point (Fig. 4) was varied (the dashed curves correspond to the nominal value $m_0 = 1$ and the solid curves 1 and 2 correspond to the realized values of $m^* = 0.9$ and $m^* = 1.1$).

As a second example, we will consider the problem of the stabilization of a mathematical pendulum in an upper unstable equilibrium position [9] (Fig. 5).

The mathematical model of such a system has the form

$$\dot{x}_1 = x_2, \ \dot{x}_2 = x_1 + x_3, \ \dot{x}_3 = u \tag{4.2}$$

where x_1 is the angle of inclination of the pendulum from the vertical, x_2 is the angular velocity of the pendulum and x_3 is the moment applied to the pendulum.

At the initial instant of time $t = 0$, system (4.2) was in the state $x_1(0) = 0.3, x_2(0) = 1.0, x_3(0) = -1.2$. It is required to stabilize it in the upper vertical position $x_1 = 0, x_2 = 0, x_3 = 0$.

The associated problem has the form

$$\begin{aligned} &\rho \to \min \\ &\dot{x}_1 = x_2, \ \dot{x}_2 = x_1 + x_3, \ \dot{x}_3 = u \\ &x_1(0) = x_1^*(\tau), \ x_2(0) = x_2^*(\tau), \ x_3(0) = x_3^*(\tau) \\ &x_1(\Theta) = 0, \ x_2(\Theta) = 0, \ x_3(\Theta) = 0 \\ &|u(t)| \leqslant \rho, \ t \in T = [0, \Theta] \end{aligned}$$

where $x^*(\tau) = (x^*_1(\tau), x^*_2(\tau), x^*_3(\tau))$ is the state of system (4.2) at the actual instant of time τ.

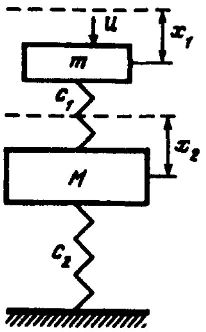

Fig. 1.

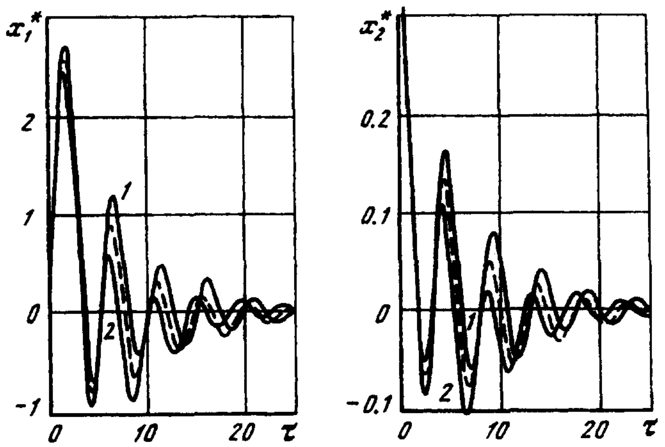

Fig. 2.

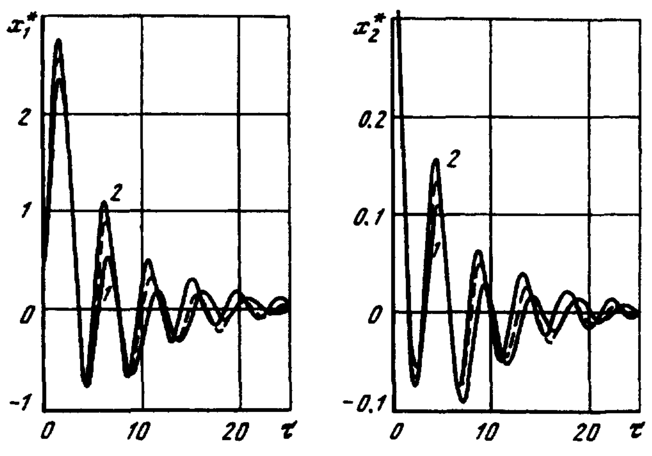

Fig. 3.

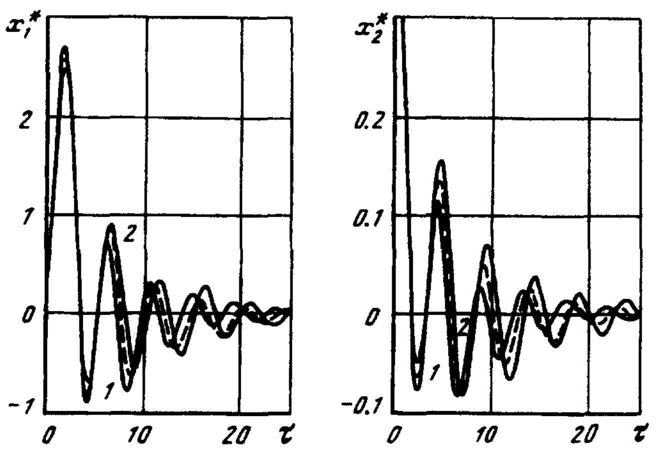

Fig. 4.

The following parameters of the method were chosen to solve the associated problem

$$\Theta = 1, \quad h = 0.025, \quad \nu = 5h$$

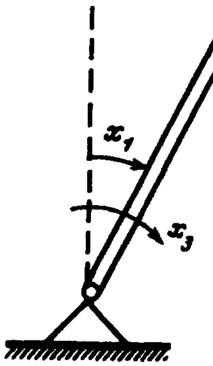

Fig. 5.

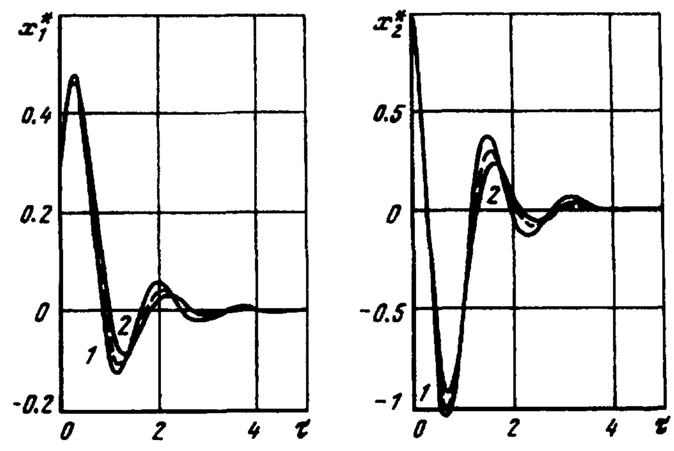

Fig. 6.

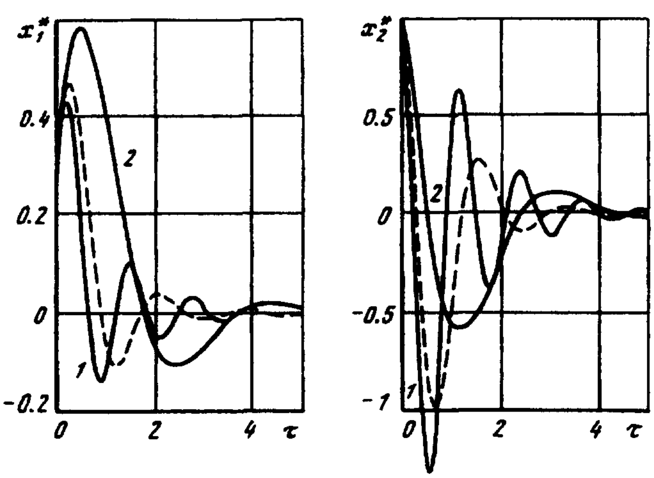

Fig. 7.

The following values of the system parameters were obtained during the operation of the stabilizer:

1. the coefficient of x_1 was varied (Fig. 6) (the dashed curves correspond to a nominal value of $1x_1$ and the solid curves 1 and 2 correspond to the realized values of $0.5x_1$ and $1.5x_1$);

2. the coefficient of x_3 was varied (Fig. 7) (the dashed curves correspond to a nominal value of $1x_3$ and the solid curves 1 and 2 correspond to the realized values of $0.5x_3$ and $1.5x_3$).

REFERENCES

1. GABASOV, R., KIRILLOVA, F. M. and KOSTRYUKOVA, O. I., Methods of stabilizing dynamic systems. *Izv. Ross. Akad. Nauk, Tekhn. Kibernetika*, 1994, 3, 67–77.
2. BALASHEVICH, N. V., GABASOV, R. and KIRILLOVA, F. M., Preset-positional optimization of dynamical systems. *Kibernetika i Sistemnyi Analiz*, 1994, 1, 66–73.
3. BARBASHIN, Ye. A., *Introduction to the Theory of Elasticity*. Nauka, Moscow, 1967.
4. BROMBERG, P. V., *Matrix Methods in the Theory of Relay and Pulsed Control*. Nauka, Moscow, 1967.
5. GABASOV, R., KIRILLOVA, F. M. and TYATYUSHKIN, A. I., *Constructive Methods of Optimization*. Part 1, *Linear Problems*. Izd. Universitetskoye, Minsk, 1984.
6. DANTZIG, G. B., *Linear Programming Extensions*. Princeton University Press, Princeton, NJ, 1963.
7. GABASOV, R. and KIRILLOVA, F. M., *Real-time construction of optimal closable feedbacks*. 13th World Congress IFAC International Federation of Automatic Control. Vol. D. San Francisco, CA. 1996, 231–236.
8. GABASOV, R., KIRILLOVA, F. M. and PRISCHEPOVA, S. V., Optimal feedback control. London: Springer, 1995, 202. *Lecture Notes in Control and Information Sciences*, **207**.
9. KRASOVSKII, N. N., Problems in the Stabilization of Controlled Motions. In MALKIN, I. G., *Theory of the Stability of Motion*. Nauka, Moscow, 1966, 475–514.

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